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Observer-based Versus Non-observer Based Adaptive Control of Electrically Driven Cooperative Manipulators Using q-analogue of the Bernstein-Schurer-Stancu Operator as Uncertainty Approximator

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Abstract: In this paper, the q-analogue of the Bernstein-Schurer-Stancu operator is proposed for uncertainty approximation in adaptive control of cooperative electrically driven manipulators. Thanks to the simplicity of the q-analogue of the Bernstein-Schurer-Stancu operator in comparison with other alternatives, the number of signals needed in the construction of the regressor vector for uncertainty approximation is reduced considerably. Studying the accuracy of model-free observers on cooperative robot manipulators is one of the motivations of this paper. Therefore, it is assumed that velocity signals are unavailable, and an observer is needed to estimate these signals. The results are also compared with the case in which velocity signals are in hand. According to the simulation result, the proposed observer can produce results that are very close to the point in which the velocity signal is used in the control law.

Keywords: Cooperative robotic system, function approximation, q-analogue of Bernstein-Schurer-Stancu operator.

1. INTRODUCTION

We have recently witnessed an increasing research interest in cooperative robot manipulators in the control community due to exciting applications in transportation, reconnaissance, search, and exploration of the environment. Compared to single manipulators, more reliability is obtained in cooperative systems thanks to the benefits of teamwork. Moreover, the industry is faced with many applications of cooperative manipulators which are out of reach of the operational capability of the single manipulator. However, the challenges of controlling single and coordinated manipulators are entirely different since cooperative robots require more variables to be considered cohesively.

Adaptive or robust control strategies are suitable options in overcomming uncertainties. Model reference adaptive control is a popular strategy. In [1], it has been applied to the control of cooperative robots. Using an adaptive controller, the influence of Markovian switched couplings in cooperative manipulators has been compensated in [2]. An adaptive controller has been designed to guarantee asymptotic convergence of the load position and boundedness of the internal forces in cooperative robots [3]. The problem of networked control of cooperative manipulators has been studied in [4] with the assumption that the dynamical parameters of the arms are not known. Thus, the controller is designed adaptively. A model reference adaptive controller has been presented in [5] for uncertain dynamical multi-agent systems. In [6], adaptive control estimates uncertainties.

Many different versions of sliding mode controllers have been presented in the literature. In [7], a second-order sliding mode controller has been proposed. In [8-10], fuzzy sliding mode controllers for cooperative robots have been introduced. A sliding mode controller based on a low-pass filter has been presented in [11]. The performance of the integral sliding mode on cooperative robot manipulators has been investigated in [12]. Also, dynamic surface control and supervisory sliding mode control have been presented in [13,14]. The combination of adaptive and robust control have also been presented for cooperative robots [15-18]. However, as mentioned above, in these robust controllers, the actuator dynamics have also been ignored.

Fuzzy systems and neural networks have also been frequently applied to robust or adaptive control of cooperative robots [19-21]. Impedance control for cooperative manipulators using neural networks has been studied in [22]. Recurrent neural networks have been used in impedance control cooperative manipulators [23]. A combination of neural networks and sliding mode has been

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proposed in [24]. An adaptive fuzzy control law is offered for the double pendulum crane with variable rope lengths [25] that can be extended to cooperative robots due to the similarity of dynamic equations. We are confronted with many tuning parameters in the controllers designed based on fuzzy systems and neural networks [26]. Moreover, the number of required signals for building the regressor vector in uncertainty estimation using fuzzy systems and neural networks is considerable [27].

To solve these shortcomings of fuzzy systems and neural networks, function approximation-based controllers using the Fourier series expansion [28], differential equations [29], and Bernstein polynomials [30] are suitable alternatives. In this paper, the q-analogue of the Bernstein-Schurer-Stancu operator is introduced as an uncertainty approximator in the adaptive control of cooperative robots. Moreover, actuator dynamics are also considered in this paper. In high-speed applications, the advantage of designing a controller by considering actuator dynamics is more tangible [31].

Many industrial robots do not provide the velocity, and force feedback; therefore, various observer-based controllers have been proposed in the literature. In [32], a robust decentralized controller for cooperative robots has been presented using a linear observer. Disturbance observer has also been utilized to control of multiple robots [33,34]. Some observers based on the sliding mode strategy have been presented in [35,36] with the aim of force estimation. In [37], an observer-based controller for cooperative robots has been given. However, information on inertia is required in this controller. In [38], a robust control scheme without velocity measurement has been presented that needs the inertia matrix to construct the observer dynamics. In [39], a model-free observer for estimating velocity signals has been presented.

In this paper, a model-free observer-controller structure is proposed for cooperative robots. Uncertainties are approximated using the q-analogue of the Bernstein-Schurer-Stancu operator, which is simpler and less computational than fuzzy systems and neural networks. The unknown parameters of the approximator will be regulated online using adaptation law. Also, the proposed observer for velocity estimation is model-free. Thus, the proposed controller is superior to most previous related works due to its model-free property in observer and controller design. Moreover, the uncertainty approximator in the proposed method is novel and more efficient than neuro/fuzzy approximators. Also, from the practical point of view, the proposed controller is more suitable due to its voltage-based structure. To highlight the satisfactory performance of the observer, a comparison with a non-observer-based controller has been presented.

The rest of this paper is organized as follows: Section 2 introduces the q-analogue of the Bernstein-Schurer-Stancu operator. Section 3 explains controller design using full-state feedback. Section 4 describes controller design using linear observers accompanied by stability analysis and performance evaluation. Section 5 presents simulation results, and Section 6 concludes the paper.

2. FUNCTION APPROXIMATION USING THE Q-ANALOUGE OF THE BERNSTEIN-SCHURER-STANCU OPERATORS

Let $\alpha, \beta, p \in \mathbb{N}^0$ (the set of all non-negative integers) be such that $0 \leq \alpha \leq \beta$, then for any $f \in C[0, 1+p]$, $q \in (0, 1)$, the q-Bernstein-Schurer-Stancu Operators are defined by [40].

$$S_{N,p}^{\alpha,\beta}(f,q,t) = \sum_{k=0}^{N+p} f\left(\frac{[k]_q+\alpha}{[N]_q+\beta}\right)\begin{bmatrix}N+p\\k\end{bmatrix}_q \times t^k(1-t)_q^{N+p-k} \forall t \in [0,\ 1], \tag{1}$$

where $\begin{bmatrix}N\\k\end{bmatrix}_q$ represents the q-binomial coefficient. In other words,

$$\begin{bmatrix}N\\k\end{bmatrix}_q = \frac{[N]_q!}{[N-k]_q![k]_q!}, \tag{2}$$

where N and k are any integers that satisfy $N \geq k \geq 0$. The q-factorial is represented by $[k]_q!$ and is described as follows:

$$[k]_q! = \begin{cases}[k]_q[k-1]_q\cdots 1, & k \geq 1,\\ 1, & k = 0,\end{cases} \tag{3}$$

where

$$[k]_q = \begin{cases}\dfrac{(1-q^k)}{(1-q)}, & q \neq 1,\\ k, & q = 1\end{cases} \tag{4}$$

denotes the q-integer of the number $k \in \mathbb{N}$. According to Theorem 1 of [40], regarding any function $f \in C[0, 1+p]$ and $0 < q < 1$, it is proved that the sequence $S_{N,p}^{\alpha,\beta}(f,q,t)$ converges uniformly to $f(t)$ on $[0, 1]$. It can be shown that

$$S_{N,p}^{\alpha,\beta}(f,q,t) = \boldsymbol{\Lambda}_f^T \boldsymbol{\xi}_f, \tag{5}$$

where

$$\boldsymbol{\Lambda}_f = \left[f\left(\tfrac{\alpha}{[N]_q+\beta}\right) \ \cdots \ f\left(\tfrac{[N+p]_q+\alpha}{[N]_q+\beta}\right)\right]^T \tag{6}$$

is a $N+p+1$ vector consisting of tunable parameters and

$$\boldsymbol{\xi}_f = \left[(1-t)_q^{N+p} \ \cdots \ t^{N+p}\right]^T \tag{7}$$

is the vector of basis functions. Equation (1) can be shown in the linear form (5), a typical adaptive control form.

3. FULL STATE FEEDBACK DESIGN

Consider the comprehensive dynamic model of m manipulators with n degree of freedom, driven by permanent magnet DC motors as follows:

$$\mathbf{M}_c(\mathbf{x}_o)\ddot{\mathbf{x}}_o + \mathbf{C}_c(\mathbf{x}_o,\dot{\mathbf{x}}_o)\dot{\mathbf{x}}_o + \mathbf{G}_c(\mathbf{x}_o) + \boldsymbol{\delta}(\mathbf{x}_o,\mathbf{v}) = \mathbf{u}, \tag{8}$$

where the parameters and symbols are exactly the same as those introduced in [41]. It can be easily shown that, the last equation can be rewritten as follows:

$$\ddot{\mathbf{x}}_o = \mathbf{u} + \boldsymbol{\sigma}(t), \tag{9}$$

where

$$\boldsymbol{\sigma}(t) = (\mathbf{M}_c^{-1}(\mathbf{x}_o) - \mathbf{I}_n)\mathbf{u} - \mathbf{M}_c^{-1}(\mathbf{C}_c\dot{\mathbf{x}}_o + \mathbf{G}_c + \boldsymbol{\delta}) \tag{10}$$

is the lumped uncertainty and $\mathbf{I}_n$ is an $n\times n$ identity matrix. Suppose that $\boldsymbol{\sigma}(t)$ is bounded, unknown, and time-varying and its variation bound is unavailable. Thus, conventional robust or adaptive control cannot be applied [42]. Attend the controller below

$$\mathbf{u} = \ddot{\mathbf{x}}_{od} - \mathbf{K}_v\dot{\mathbf{e}}_o - \mathbf{K}_p\mathbf{e}_o - \hat{\boldsymbol{\sigma}}(t), \tag{11}$$

where $0 < \mathbf{K}_p \in \Re^{n\times n}$ and $0 < \mathbf{K}_v \in Re^{n\times n}$ are constant gain matrices, $\hat{\boldsymbol{\sigma}}(t)$ is an approximation of $\boldsymbol{\sigma}(t)$ and $\mathbf{e}_o = \mathbf{x}_o - \mathbf{x}_{od}$ declares tracking error for the object position/orientation.

Assumption 1: The desired trajectory is smooth in the sense that is bounded and uniformly continuous. Moreover, up to a necessary order, its derivatives are bounded and uniformly continuous.

Replacing $\mathbf{u}$ in (9) with (11) gives

$$\ddot{\mathbf{e}}_o + \mathbf{K}_v\dot{\mathbf{e}}_o + \mathbf{K}_p\mathbf{e}_o = \boldsymbol{\sigma}(t) - \hat{\boldsymbol{\sigma}}(t). \tag{12}$$

Define

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_n & \mathbf{I}_n \\ -\mathbf{K}_p & -\mathbf{K}_v \end{bmatrix} \in \Re^{2n\times 2n},\ \mathbf{B} = \begin{bmatrix} \mathbf{0}_n \\ \mathbf{I}_n \end{bmatrix} \in \Re^{2n\times n},$$
$$\mathbf{x} = \begin{bmatrix} \mathbf{e}_o \\ \dot{\mathbf{e}}_o \end{bmatrix} \in \Re^{2n}.$$

Thus, (12) is given by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}(\boldsymbol{\sigma}(t) - \hat{\boldsymbol{\sigma}}(t)). \tag{13}$$

Converging $\mathbf{x}(t)$ to zero can result from (13) if an adaptation rule is designed so that $\hat{\boldsymbol{\sigma}}(t) \to \boldsymbol{\sigma}(t)$. With this in mind, the q-analogue of the Bernstein-Schurer-Stancu operator is used to represent $\boldsymbol{\sigma}(t)$ as a linear mixture of basis functions as

$$\boldsymbol{\sigma}(t) = \mathbf{W}_\sigma^T\mathbf{Z}_\sigma + \boldsymbol{\varepsilon}_\sigma, \tag{14}$$

where $\mathbf{W}_\sigma \in \Re^{n\beta_1}$ is q-analogue of the Bernstein-Schurer-Stancu operator coefficient, β_1 is the basis function number, $\mathbf{Z}_\sigma \in \Re^{n\beta_1}$ is the regressor vector, and $\boldsymbol{\varepsilon}_\sigma$ is the reconstruction error vector of $\boldsymbol{\sigma}(t)$. Based on the q-analogue of the Bernstein-Schurer-Stancu operator, the corresponding estimates can then be given by the equation below

$$\hat{\boldsymbol{\sigma}}(t) = \hat{\mathbf{W}}_\sigma^T\mathbf{Z}_\sigma, \tag{15}$$

where $\hat{\mathbf{W}}_\sigma \in \Re^{n\beta_1\times n}$ is the estimation of $\mathbf{W}_\sigma$. Now, by substituting (14) and (15) into (13), we have

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}(\tilde{\mathbf{W}}_\sigma^T\mathbf{Z}_\sigma + \boldsymbol{\varepsilon}_\sigma), \tag{16}$$

in which $\tilde{\mathbf{W}}_\sigma = \mathbf{W}_\sigma - \hat{\mathbf{W}}_\sigma$ is the parametric error of the q-analogue of the Bernstein-Schurer-Stancu operator coefficients.

Remark 1: It is important to note that $\mathbf{v}$ is the controller output that will be assigned to the robotic system. Consequently, the control law that is responsible for the regulation of the position is defined as

$$\mathbf{v} = \mathbf{J}_e^T(\mathbf{q})\mathbf{F}_{Ic} + \mathbf{J}_e^T(\mathbf{q})(\mathbf{J}_o^T(\mathbf{x}_o))^\dagger\mathbf{u}, \tag{17}$$

where the internal force control term $\mathbf{F}_{Ic} \in \Re^{mn}$ is assigned as

$$\mathbf{F}_{Ic} = \mathbf{K}_I(\mathbf{F}_{Id} - \mathbf{F}_I) + \mathbf{F}_{Id}, \tag{18}$$

with $\mathbf{K}_I \in \Re^{mn\times mn}$ representing a positive diagonal gain matrix. It is essential to mention that $\mathbf{F}_{Ic}$ is in the null space of $\mathbf{J}_o^T(\mathbf{x}_o)$, since $\mathbf{F}_{Id}$ has that property.

The following theorem presents the primary outcomes of this paper.

Theorem 1: Consider the dynamic system in (9). Assume the described controller in (11), in which the lumped uncertainty is approximated by (15). Using σ-modification, we have

$$\dot{\hat{\mathbf{W}}}_\sigma = \mathbf{Q}_\sigma^{-1}(\mathbf{Z}_\sigma\mathbf{x}^T\boldsymbol{\Xi}\mathbf{B} - \kappa_\sigma\hat{\mathbf{W}}_\sigma), \tag{19}$$

where κ_σ is a positive scalar and $\mathbf{Q}_\sigma \in \Re^{n\beta_1\times n\beta_1} > 0$ shows its learning rate. The vectors $\mathbf{x}(t)$ and $\hat{\mathbf{W}}_\sigma$ are then uniformly ultimately bounded, if the number of regressor terms, control gains, and other design parameters are appropriately selected.

Proof: Consider the following function

$$V(\mathbf{x},\hat{\mathbf{W}}_\sigma) = \mathbf{x}^T\boldsymbol{\Xi}\mathbf{x} + Tr(\tilde{\mathbf{W}}_\sigma^T\mathbf{Q}_\sigma\tilde{\mathbf{W}}_\sigma), \tag{20}$$

where $Tr(\cdot)$ is the trace operator. The time derivative of (20) accompanying (16) and (19) leads to

$$\dot{V}(\mathbf{x},\tilde{\mathbf{W}}_\sigma) = \mathbf{x}^T(\mathbf{A}^T\boldsymbol{\Xi} + \boldsymbol{\Xi}\mathbf{A})\mathbf{x} + 2\mathbf{x}^T\boldsymbol{\Xi}\mathbf{B}\boldsymbol{\varepsilon}_\sigma + 2\kappa_\sigma Tr(\tilde{\mathbf{W}}_\sigma^T\hat{\mathbf{W}}_\sigma). \tag{21}$$

Since $\mathbf{A}$ is Hurwitz; it is concluded that

$$\mathbf{A}^T\boldsymbol{\Xi} + \boldsymbol{\Xi}\mathbf{A} + \mathbf{Q} = \mathbf{0}, \tag{22}$$

where $\boldsymbol{\Xi} \in \Re^{2n\times 2n}$ and $\mathbf{Q} \in \Re^{2n\times 2n}$ are symmetric positive-definite matrices. Therefore, (21) is simplified to

$$\dot{V}(\mathbf{x},\tilde{\mathbf{W}}_\sigma) = -\mathbf{x}^T\mathbf{Q}\mathbf{x} + 2\mathbf{x}^T\boldsymbol{\Xi}\mathbf{B}\boldsymbol{\varepsilon}_\sigma + 2\kappa_\sigma Tr(\tilde{\mathbf{W}}_\sigma^T\hat{\mathbf{W}}_\sigma). \tag{23}$$

Result 1: Suppose that $\boldsymbol{\varepsilon}_\sigma$ is negligible. Then, the σ-modification term in (19) is not needed. Therefore, (23) is converted to

$$\dot{V}(\mathbf{x},\tilde{\mathbf{W}}_\sigma) \le -\mathbf{x}^T\mathbf{Q}\mathbf{x}.$$

In order to verify the asymptotic convergence of $\mathbf{x}$, the Barbalat's Lemma can be used.

Result 2: Suppose that $\boldsymbol{\varepsilon}_\sigma$ is large. It is easy to prove the following inequalities hold

$$\begin{aligned}&-\mathbf{x}^T\mathbf{Q}\mathbf{x} + 2\mathbf{x}^T\boldsymbol{\Xi}\mathbf{B}\boldsymbol{\varepsilon}_\sigma\\ &\le -\frac{1}{2}\underline{\lambda}(\mathbf{Q})\|\mathbf{x}\|^2 + \frac{2\overline{\sigma}^2(\boldsymbol{\Xi}\mathbf{B})}{\underline{\lambda}(\mathbf{Q})}\|\varepsilon_\sigma\|^2, \qquad (24)\\ &2Tr(\tilde{\mathbf{W}}_\sigma^T\hat{\mathbf{W}}_\sigma) \le Tr(\mathbf{W}_\sigma^T\mathbf{W}_\sigma) - Tr(\tilde{\mathbf{W}}_\sigma^T\tilde{\mathbf{W}}_\sigma), \qquad (25)\end{aligned}$$

where $\overline{\sigma}(\boldsymbol{\Xi}\mathbf{B})$ represent the maximum singular value of $\boldsymbol{\Xi}\mathbf{B}$. Together with these relationships and

$$V(\mathbf{x},\tilde{\mathbf{W}}_\sigma) \le \overline{\lambda}(\boldsymbol{\Xi})\|\mathbf{x}\|^2 + \overline{\lambda}(\mathbf{Q}_\sigma)Tr(\tilde{\mathbf{W}}_\sigma^T\tilde{\mathbf{W}}_\sigma). \tag{26}$$

One can write (23) in the form of

$$\begin{aligned}\dot{V} \le& -\mu V + \left(\mu\overline{\lambda}(\boldsymbol{\Xi}) - \frac{1}{2}\underline{\lambda}(\mathbf{Q})\right)\|\mathbf{x}\|^2\\ &+ \kappa_\sigma Tr(\mathbf{W}_\sigma^T\mathbf{W}_\sigma) + (\mu\overline{\lambda}(\mathbf{Q}_\sigma) - \kappa_\sigma)Tr(\tilde{\mathbf{W}}_\sigma^T\tilde{\mathbf{W}}_\sigma)\\ &+ \frac{2\overline{\sigma}^2(\boldsymbol{\Xi}\mathbf{B})}{\underline{\lambda}(\mathbf{Q})}\|\boldsymbol{\varepsilon}_\sigma\|^2.\end{aligned} \tag{27}$$

By selecting

$$\mu \le \min\left\{\frac{\underline{\lambda}(\mathbf{Q})}{2\overline{\lambda}(\boldsymbol{\Xi})}, \frac{\kappa_\sigma}{\overline{\lambda}(\mathbf{Q}_\sigma)}\right\}. \tag{28}$$

Equation (27) can be further derived as

$$\begin{aligned}\dot{V}(\mathbf{x},\tilde{\mathbf{W}}_\sigma) \le& -\mu V + \frac{2\overline{\sigma}^2(\boldsymbol{\Xi}\mathbf{B})}{\underline{\lambda}(\mathbf{Q})}\|\varepsilon_\sigma\|^2\\ &+ \kappa_\sigma Tr(\mathbf{W}_\sigma^T\mathbf{W}_\sigma).\end{aligned} \tag{29}$$

As a result $\dot{V} < 0$, whenever

$$V > \frac{2\overline{\sigma}^2(\boldsymbol{\Xi}\mathbf{B})}{\mu\underline{\lambda}(\mathbf{Q})}\sup_{t_0<\tau}\|\varepsilon_\sigma(\tau)\|^2 + \frac{\kappa_\sigma}{\mu}Tr(\mathbf{W}_\sigma^T\mathbf{W}_\sigma). \tag{30}$$

Hence, we have proved that $\mathbf{x}$ and $\tilde{\mathbf{W}}_\sigma$ are uniformly ultimately bounded. □

In order to verify the boundedness of the internal force errors, consider (31), which is obtained by (8), (11), and (17).

$$\begin{aligned}&\mathbf{F}_{Id} - \mathbf{F}_I\\ &= (\mathbf{I}_n + \mathbf{K}_I)^{-1}\left\{(\mathbf{J}_o^T)^\dagger(\boldsymbol{\sigma} - \mathbf{u} + \mathbf{M}_c\ddot{\mathbf{x}}_o + \mathbf{C}_c\dot{\mathbf{x}}_o + \mathbf{G}_c)\right\}.\end{aligned} \tag{31}$$

The force error is reversely related to $(\mathbf{I}_n + \mathbf{K}_I)$ and bounded. It can be suppressed to the desired value by choosing enough big value for $\mathbf{K}_I$ gain.

3.1. Performance evaluation

In the previous subsection, the boundedness of the closed-loop signals was verified. It is to be noted that considering practical issues in implantations; transient performance fulfillment is also a tremendous key issue. Equation (29) provides

$$\begin{aligned}V(t) \le& V(t_0)e^{-\mu(t-t_0)} + \frac{2\overline{\sigma}^2(\boldsymbol{\Xi}\mathbf{B})}{\mu\underline{\lambda}(\mathbf{Q})}\sup_{t_0<\tau<t}\|\boldsymbol{\varepsilon}_\sigma(\tau)\|^2\\ &+ \frac{\kappa_\sigma}{\mu}Tr(\mathbf{W}_\sigma^T\mathbf{W}_\sigma).\end{aligned} \tag{32}$$

Using (20), we have

$$\|\mathbf{x}\| \le \sqrt{\frac{V}{\underline{\lambda}(\boldsymbol{\Xi})}}. \tag{33}$$

This leaads to

$$\begin{aligned}\|\mathbf{x}\| \le& \sqrt{\frac{V(t_0)}{\underline{\lambda}(\boldsymbol{\Xi})}}e^{-0.5\mu(t-t_0)} + \sqrt{\frac{\kappa_\sigma}{\mu\underline{\lambda}(\boldsymbol{\Xi})}Tr(\mathbf{W}_\sigma^T\mathbf{W}_\sigma)}\\ &+ \sqrt{\frac{2}{\mu\underline{\lambda}(\boldsymbol{\Xi})\underline{\lambda}(\mathbf{Q})}}\overline{\sigma}(\boldsymbol{\Xi}\mathbf{B})\sup_{t_0<\tau<t}\|\boldsymbol{\varepsilon}_\sigma(\tau)\|,\end{aligned} \tag{34}$$

that guarantees $\|\mathbf{x}\|$ is bounded. Inequality (34) shows that by tuning some gains, one can enhance the reduction rate of $\|\mathbf{x}\|$. Therefore,

$$\begin{aligned}\lim_{t\to\infty}\|\mathbf{x}(t)\| \le& \sqrt{\frac{2}{\mu\underline{\lambda}(\boldsymbol{\Xi})\underline{\lambda}(\mathbf{Q})}}\overline{\sigma}(\boldsymbol{\Xi}\mathbf{B})\sup_{t_0<\tau<t}\|\varepsilon_\sigma(\tau)\|\\ &+ \sqrt{\frac{\kappa_\sigma}{\mu\underline{\lambda}(\boldsymbol{\Xi})}Tr(\mathbf{W}_\sigma^T\mathbf{W}_\sigma)}.\end{aligned} \tag{35}$$

The result is fulfilling the analysis of performance transient.

4. CONTROLLER DESIGN USING LINEAR STATE ESTIMATED FEEDBACK

The proposed control input and observer dynamics are as follows:

$$\mathbf{u} = \ddot{\mathbf{x}}_{od} - \mathbf{K}_v\dot{\hat{\mathbf{e}}}_o - \mathbf{K}_p\hat{\mathbf{e}}_o - \hat{\boldsymbol{\sigma}}(t), \tag{36}$$

$$\begin{cases}\dot{\hat{\mathbf{e}}}_o = \boldsymbol{\rho} + \mathbf{L}_v(\mathbf{e}_o - \hat{\mathbf{e}}_o),\\ \dot{\rho} = \mathbf{L}_p(\mathbf{e}_o - \hat{\mathbf{e}}_o),\end{cases} \tag{37}$$

where $0 < \mathbf{K} \in \Re$ and $0 < \mathbf{K}_p \in \Re^{n\times n}$ are gain matrices; $\hat{\mathbf{e}}_o = \hat{\mathbf{x}}_o - \mathbf{x}_{od}$ is the estimation of $\mathbf{e}_o(t)$; $\mathbf{L}_v \in \Re^{n\times n}$ and

$\mathbf{L}_p \in \Re^{ntimesn}$ are diagonal and positive observer gain matrices and $\hat{\boldsymbol{\sigma}}(t)$ is defined as before. Let us make the following assumption on the structure of $\mathbf{K}_p$, $\mathbf{K}_v$, $\mathbf{L}_v$, and $\mathbf{L}_p$.

Assumption 2: $\mathbf{K}_p$, $\mathbf{K}_v$, $\mathbf{L}_v$, and $\mathbf{L}_p$ satisfy the following equations:

$$\mathbf{K}_p = \mu \mathbf{K}_v, \tag{38}$$
$$\mathbf{K}_v = (k_v + \gamma)\mathbf{I}_n, \tag{39}$$
$$\mathbf{L}_p = \mu l_v \mathbf{I}_n, \tag{40}$$
$$\mathbf{L}_v = (l_v + \mu)\mathbf{I}_n, \tag{41}$$

where μ, k_v, γ, and l_v are positive constants and $k_v > \mu$. Substituting (36) into (9) and using definition (38) yields

$$\ddot{\mathbf{e}}_o + \mathbf{K}_v \dot{\hat{\mathbf{e}}}_o + \mu \mathbf{K}_v \hat{\mathbf{e}}_o = \boldsymbol{\sigma}(t) - \hat{\boldsymbol{\sigma}}(t). \tag{42}$$

Define

$$\mathbf{s}_1 = \dot{\mathbf{x}}_o - \dot{\mathbf{x}}_{or}, \tag{43}$$
$$\mathbf{s}_2 = \dot{\mathbf{x}}_o - \dot{\mathbf{x}}_{o\ell}, \tag{44}$$

where

$$\tilde{\mathbf{x}}_o = \mathbf{e}_o - \hat{\mathbf{e}}_o \tag{45}$$

denotes the task-space position estimation error. Then, the closed-loop error dynamics are given by

$$\ddot{\mathbf{e}}_o = -\mathbf{K}_v \mathbf{s}_1 + \mathbf{K}_v \mathbf{s}_2 + \boldsymbol{\sigma}(t) - \hat{\boldsymbol{\sigma}}(t), \tag{46}$$
$$\dot{\mathbf{s}}_2 + (l_v \mathbf{I}_n - \mathbf{K}_v)\mathbf{s}_2 = -\mathbf{K}_v \mathbf{s}_1 + \tilde{\mathbf{W}}_\sigma \mathbf{Z}_\sigma + \boldsymbol{\varepsilon}_\sigma, \tag{47}$$

where we have utilized (44) and the following equality

$$\ddot{\mathbf{x}}_{o\ell}(t) = \ddot{\mathbf{x}}_{od}(t) = l_v \mathbf{s}_2(t). \tag{48}$$

Utilizing the procedure as same as those done in the previous section, we present $\hat{\boldsymbol{\sigma}}(t)$ and $\boldsymbol{\sigma}(t)$ by (15) and (14), respectively, with the same parameters' definitions and dimensions. Substituting (14) and (15) into (46) and (47) yields the output tracking loop dynamics as follows:

$$\ddot{\mathbf{e}}_o = -\mathbf{K}_v \mathbf{s}_1 + \mathbf{K}_v \mathbf{s}_2 + \tilde{\mathbf{W}}_\sigma \mathbf{Z}_\sigma + \boldsymbol{\varepsilon}_\sigma, \tag{49}$$
$$\dot{\mathbf{s}}_2 + (l_v \mathbf{I}_n - \mathbf{K}_v)\mathbf{s}_2 = -\mathbf{K}_v \mathbf{s}_1 + \tilde{\mathbf{W}}_\sigma \mathbf{Z}_\sigma + \boldsymbol{\varepsilon}_\sigma, \tag{50}$$

where $\tilde{\mathbf{W}}_\sigma = \mathbf{W}_\sigma - \hat{\mathbf{W}}_\sigma$ is the parametric error related to the q-analogue of the Bernstein-Schurer-Stancu operator's coefficients.

4.1. Stability analysis

To investigate the system stability, a Lyapunov-like candidate function is introduced as follows:

$$V(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) = \frac{1}{2}\mathbf{y}^T \mathbf{P} \mathbf{y} + \frac{1}{2} Tr(\tilde{\mathbf{W}}_\sigma^T \boldsymbol{\Gamma}_\sigma \tilde{\mathbf{W}}_\sigma), \tag{51}$$

where

$$\mathbf{y} = \left[\dot{\mathbf{e}}_o^T \quad (\mu \mathbf{e}_o)^T \quad \dot{\tilde{\mathbf{x}}}_o^T \quad (\mu \tilde{\mathbf{x}}_o)^T\right]^T \in \Re^{4n}, \tag{52}$$

$$\mathbf{P} = \begin{bmatrix} \begin{bmatrix} \mathbf{I}_n & \mathbf{I}_n \\ \mathbf{I}_n & 2\mu^{-1}k_v \mathbf{I}_n \end{bmatrix} & \mathbf{0}_{2n} \\ \mathbf{0}_{2n} & \begin{bmatrix} \mathbf{I}_n & \mathbf{I}_n \\ \mathbf{I}_n & 2\mu^{-1}k_v \mathbf{I}_n + \mathbf{I}_n \end{bmatrix} \end{bmatrix} \in \Re^{4n \times 4n}, \tag{53}$$

with $\mathbf{0}_{2n}$ denoting the null matrix of dimension $2n$ and $\boldsymbol{\Gamma}_\sigma \in \Re^{n\beta_1 \times n\beta_1}$ is a positive-definite matrix.

Lemma 1: The upper and lower bounds of (51) are as follows:

$$V(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) \leq \frac{1}{2}\overline{\lambda}(\mathbf{P})\|\mathbf{y}\|^2 + \frac{1}{2}\overline{\lambda}(\boldsymbol{\Gamma}_\sigma) Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma), \tag{54}$$

$$V(\mathbf{y}, \tilde{\mathbf{W}}_\sigma) \geq \frac{1}{2}\underline{\lambda}(\mathbf{P})\|\mathbf{y}\|^2 + \frac{1}{2}\underline{\lambda}(\boldsymbol{\Gamma}_\sigma) Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma), \tag{55}$$

with $\overline{\lambda}(\mathbf{P})$ and $\underline{\lambda}(\mathbf{P})$ defined as

$$\overline{\lambda}(\mathbf{P}) = 6\mu^{-1}k_v, \ \underline{\lambda}(\mathbf{P}) = \frac{1}{3}. \tag{56}$$

Proof: Introducing

$$\mathbf{z}^T = \left[\mathbf{s}_1^T \quad (\mu \mathbf{e}_o)^T \quad \mathbf{s}_2^T \quad (\mu \tilde{\mathbf{x}}_o)^T\right]. \tag{57}$$

Function (51) could be rewritten as

$$V(\mathbf{z}, \tilde{\mathbf{W}}_\sigma) = \frac{1}{2}\mathbf{z}^T \boldsymbol{\Lambda} \mathbf{z} + \frac{1}{2} Tr(\tilde{\mathbf{W}}_\sigma^T \boldsymbol{\Gamma}_\sigma \tilde{\mathbf{W}}_\sigma), \tag{58}$$

where

$$\boldsymbol{\Lambda} = \begin{bmatrix} \begin{bmatrix} \mathbf{I}_n & \mathbf{0}_n \\ \mathbf{0}_n & 2\mu^{-1}k_v \mathbf{I}_n - \mathbf{I}_n \end{bmatrix} & \mathbf{0}_{2n} \\ \mathbf{0}_{2n} & \begin{bmatrix} \mathbf{I}_n & \mathbf{0}_n \\ \mathbf{0}_n & 2\mu^{-1}k_v \mathbf{I}_n \end{bmatrix} \end{bmatrix} \in \Re^{4n \times 4n}. \tag{59}$$

It is not difficult to demonstrate that

$$\begin{aligned} &\frac{1}{2}(\underline{\lambda}(\boldsymbol{\Lambda})\|\mathbf{z}\|^2 + \underline{\lambda}(\boldsymbol{\Gamma}_\sigma) Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma)) \\ &\leq V(\mathbf{z}, \tilde{\mathbf{W}}_\sigma) \\ &\leq \frac{1}{2}(\bar{\lambda}(\boldsymbol{\Lambda})\|\mathbf{z}\|^2 + \bar{\lambda}(\boldsymbol{\Gamma}_\sigma) Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma)), \end{aligned} \tag{60}$$

where

$$\underline{\lambda}(\boldsymbol{\Lambda}) = 1, \ \overline{\lambda}(\boldsymbol{\Lambda}) = 2\mu^{-1}k_v. \tag{61}$$

By definition

$$\mathbf{z} = \mathbf{T}\mathbf{y}, \tag{62}$$

using (52) and

$$\mathbf{T} = Blockdiag\left(\begin{bmatrix} \mathbf{I}_n & \mathbf{I}_n \\ \mathbf{0}_n & \mathbf{I}_n \end{bmatrix}, \begin{bmatrix} \mathbf{I}_n & \mathbf{I}_n \\ \mathbf{0}_n & \mathbf{I}_n \end{bmatrix}\right)$$

$$\in \Re^{4n\times 4n}. \tag{63}$$

It can be easily shown that

$$\frac{1}{3}\|\mathbf{y}\|^2 \leq \|\mathbf{z}\|^2 \leq 3\|\mathbf{y}\|^2. \tag{64}$$

Together with (60) and (61), it implies (54)-(56). □

Differentiating (51) concerning time; using (39), (49), (50), and rearranging with some mathematical simplification, we have

$$\begin{aligned}\dot{V}(\mathbf{y},\tilde{\mathbf{W}}_\sigma) = &-\dot{\mathbf{e}}_o^T k_v \mathbf{I}_n \dot{\mathbf{e}}_o - \mathbf{e}_o^T \mu k_v \mathbf{I}_n \mu \mathbf{e}_o + \dot{\mathbf{e}}_o^T \mu \mathbf{I}_n \dot{\mathbf{e}}_o \\ &- \dot{\tilde{\mathbf{x}}}_o^T k_v \mathbf{I}_n \dot{\tilde{x}}_o - \tilde{\mathbf{x}}_o^T \mu k_v \mathbf{I}_n \mu \tilde{\mathbf{x}}_o - \mathbf{s}_1^T \gamma \mathbf{I}_n \mathbf{s}_1 \\ &- \mathbf{s}_2^T \gamma \mathbf{I}_n \mathbf{s}_2 - \mathbf{s}_2^T (l_d \mathbf{I}_n - 2\mathbf{K}_v)\mathbf{s}_2 \\ &+ (\mathbf{s}_1^T + \mathbf{s}_2^T)\boldsymbol{\varepsilon}_\sigma + (\mathbf{s}_1^T + \mathbf{s}_2^T)\tilde{\mathbf{W}}_\sigma^T \mathbf{Z}_\sigma \\ &- Tr(\tilde{\mathbf{W}}_\sigma^T \boldsymbol{\Gamma}_\sigma \dot{\hat{\mathbf{W}}}_\sigma),\end{aligned} \tag{65}$$

or equivalently

$$\begin{aligned}\dot{V}(\mathbf{y},\tilde{\mathbf{W}}_\sigma) = &-\mathbf{y}^T \mathbf{Q}_o \mathbf{y} - \mathbf{s}_1^T \gamma \mathbf{I}_n \mathbf{s}_1 - \mathbf{s}_2^T \gamma \mathbf{I}_n \mathbf{s}_2 \\ &- \mathbf{s}_2^T (l_v \mathbf{I}_n - 2\mathbf{K}_v)\mathbf{s}_2 + (\mathbf{s}_1^T + \mathbf{s}_2^T)\boldsymbol{\varepsilon}_\sigma \\ &- Tr(\tilde{\mathbf{W}}_\sigma^T(\boldsymbol{\Gamma}_\sigma \dot{\hat{\mathbf{W}}}_\sigma - \mathbf{Z}_\sigma(\mathbf{s}_1^T + \mathbf{s}_2^T))),\end{aligned} \tag{66}$$

where

$$\begin{aligned}\mathbf{Q}_o = &\, blockdiag\,[(k_v - \mu)\mathbf{I}_n,\ k_v \mathbf{I}_n,\ k_v \mathbf{I}_n,\ k_v \mathbf{I}_n] \\ &\in \Re^{4n\times 4n}.\end{aligned} \tag{67}$$

By selecting the updated laws as

$$\begin{aligned}\hat{\mathbf{W}}_\sigma &= \boldsymbol{\Gamma}_\sigma^{-1}\mathbf{Z}_\sigma(t)\aleph^T(t) + \boldsymbol{\Delta}(t), \\ \dot{\boldsymbol{\Delta}}(t) &= -\boldsymbol{\Gamma}_\sigma^{-1}\dot{\mathbf{Z}}_\sigma(t)\aleph^T(t) + \mu \boldsymbol{\Gamma}_\sigma^{-1}\mathbf{Z}_\sigma(t)\aleph^T(t) \\ &\quad - \overline{\kappa}_\sigma \boldsymbol{\Gamma}_\sigma^{-1}\hat{\mathbf{W}}_\sigma(t),\end{aligned} \tag{68}$$

and

$$\aleph(t) = \mathbf{e}_o(t) + \tilde{\mathbf{x}}_o(t), \tag{69}$$

the inequality (66) is simplified as

$$\begin{aligned}\dot{V}(\mathbf{y},\tilde{\mathbf{W}}_\sigma) \leq &-\underline{\lambda}(\mathbf{Q}_o)\|\mathbf{y}\|^2 - \gamma\|\mathbf{s}_1\|^2 - \gamma\|\mathbf{s}_2\|^2 \\ &+ \|\mathbf{s}_1\|\|\boldsymbol{\varepsilon}_\sigma\| + \|\mathbf{s}_2\|\|\boldsymbol{\varepsilon}_\sigma\| \\ &+ \overline{\kappa}_\sigma Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma).\end{aligned} \tag{70}$$

where inequality $l_v\mathbf{I}_n - 2\mathbf{K}_v > 0$ has been used. Utilizing the inequality $\|\mathbf{s}_1\|, \|\mathbf{s}_2\| \leq \|\mathbf{z}\| \leq \sqrt{3}\|\mathbf{y}\|$, (70) is given by

$$\begin{aligned}\dot{V}(\mathbf{y},\tilde{\mathbf{W}}_\sigma) \leq &-\underline{\lambda}(\mathbf{Q}_o)\|\mathbf{y}\|^2 + 2\sqrt{3}\|\mathbf{y}\|\|\boldsymbol{\varepsilon}_\sigma\| \\ &+ \overline{\kappa}_\sigma Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma).\end{aligned} \tag{71}$$

Result 3: Assume that enough basis functions are utilized in the regressor vector such that the estimation error is tinny. Therefore,

$$\dot{V}(\mathbf{y},\tilde{\mathbf{W}}_\sigma) \leq -\underline{\lambda}(\mathbf{Q}_o)\|\mathbf{y}\|^2. \tag{72}$$

Thus, $\mathbf{y}$ asymptotically converges to zero.

Result 4: Assume that in (71) is large. Then, we can write

$$\begin{aligned}&-\underline{\lambda}(\mathbf{Q}_o)\|\mathbf{y}\|^2 + 2\sqrt{3}\|\mathbf{y}\|\|\boldsymbol{\varepsilon}_\sigma\| \\ &\leq -0.5\underline{\lambda}(\mathbf{Q}_o)\|\mathbf{y}\|^2 + \frac{6}{\underline{\lambda}(\mathbf{Q}_o)}\|\boldsymbol{\varepsilon}_\sigma\|^2.\end{aligned} \tag{73}$$

Regarding this equation and using (25); (71) is rewritten as

$$\begin{aligned}\dot{V}(\mathbf{y},\tilde{\mathbf{W}}_\sigma) \leq &-0.5\underline{\lambda}(\mathbf{Q}_o)\|\mathbf{y}\|^2 - 0.5\overline{\kappa}_\sigma Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma) \\ &+ \frac{6}{\underline{\lambda}(\mathbf{Q}_o)}\|\boldsymbol{\varepsilon}_\sigma\|^2 + 0.5\overline{\kappa}_\sigma Tr(\mathbf{W}_\sigma^T \mathbf{W}_\sigma).\end{aligned} \tag{74}$$

Considering (54), one can easily relate (74) to as follows:

$$\begin{aligned}\dot{V}(\mathbf{y},\tilde{\mathbf{W}}_\sigma) \leq &-\omega V + 0.5(\omega\overline{\lambda}(\mathbf{P}) - \underline{\lambda}(\mathbf{Q}_o))\|\mathbf{y}\|^2 \\ &+ 0.5(\omega\overline{\lambda}(\boldsymbol{\Gamma}_\sigma) - \overline{\kappa}_\sigma)Tr(\tilde{\mathbf{W}}_\sigma^T \tilde{\mathbf{W}}_\sigma) \\ &+ \frac{6}{\underline{\lambda}(\mathbf{Q}_o)}\|\boldsymbol{\varepsilon}_\sigma\|^2 + 0.5\overline{\kappa}_\sigma Tr(\mathbf{W}_\sigma^T \mathbf{W}_\sigma).\end{aligned} \tag{75}$$

Now, by choosing ω as

$$\omega \leq \min\left\{\frac{\underline{\lambda}(\mathbf{Q}_o)}{\overline{\lambda}(\mathbf{P})}, \frac{\overline{\kappa}_\sigma}{\overline{\lambda}(\boldsymbol{\Gamma}_\sigma)}\right\}. \tag{76}$$

Equation (75) is simplified to

$$\begin{aligned}\dot{V}(\mathbf{y},\tilde{\mathbf{W}}_\sigma) \leq &-\omega V + 0.5\overline{\kappa}_\sigma Tr(\mathbf{W}_\sigma^T \mathbf{W}_\sigma) \\ &+ \frac{6}{\underline{\lambda}(\mathbf{Q}_o)}\|\boldsymbol{\varepsilon}_\sigma\|^2.\end{aligned} \tag{77}$$

Thus, if $(\mathbf{y}, \tilde{\mathbf{W}}_\sigma)$ belongs to the following set, $\dot{V}(\mathbf{y}, \tilde{\mathbf{W}}_\sigma)$ is negative-definite.

$$\begin{aligned}&(\mathbf{y},\tilde{\mathbf{W}}_\sigma) \\ &\in \Omega \equiv \left\{(\mathbf{y},\tilde{\mathbf{W}}_\sigma) \mid V > \frac{6}{\omega\underline{\lambda}(\mathbf{Q}_o)}\sup_{t_0\leq\tau}\|\boldsymbol{\varepsilon}_\sigma(\tau)\|^2\right. \\ &\qquad\qquad \left. + \frac{\overline{\kappa}_\sigma}{2\omega}Tr(\mathbf{W}_\sigma^T \mathbf{W}_\sigma)\right\}.\end{aligned} \tag{78}$$

Therefore, the uniform ultimate boundedness of $\mathbf{y}$ and $\tilde{\mathbf{W}}_\sigma$ is proven. Moreover, according to (78), the observer error can become smaller by choosing proper values for ω, $\overline{\kappa}_\sigma$, and $\mathbf{Q}_o$. It should be considered that the internal force boundedness errors can be checked out just like before.

4.2. Performance evaluation

To show the satisfactory performance of the proposed controller-observer structure in the transient state, (77) is solved to obtain

$$V(t) \leq V(t_0)e^{-\omega(t-t_0)} + \frac{6}{\omega\underline{\lambda}(\mathbf{Q}_o)}\sup_{t_0<\tau<t}\|\boldsymbol{\varepsilon}_\sigma(\tau)\|^2$$

$$+\frac{\overline{\kappa}_\sigma}{2\omega}Tr(\mathbf{W}_\sigma^T\mathbf{W}_\sigma). \tag{79}$$

Considering (55), one can conclude that

$$\|\mathbf{y}\| \le \sqrt{\frac{2V}{\underline{\lambda}(\mathbf{P})}}. \tag{80}$$

That is simplified to (81) by employing (79)

$$\begin{aligned}\|\mathbf{y}\| \le & \sqrt{\frac{2V(t_0)}{\underline{\lambda}(\mathbf{P})}}e^{-0.5\omega(t-t_0)} \\ & + \sqrt{\frac{12}{\omega\underline{\lambda}(\mathbf{P})\underline{\lambda}(\mathbf{Q}_o)}} \sup_{t_0<\tau<t}\|\boldsymbol{\varepsilon}_\sigma(\tau)\| \\ & + \sqrt{\frac{\overline{\kappa}_\sigma}{\omega\underline{\lambda}(\mathbf{P})}Tr(\mathbf{W}_\sigma^T\mathbf{W}_\sigma)}. \end{aligned} \tag{81}$$

It is concluded from (81) that the time history of is bounded by an exponential function plus some constant. This also implies that we may improve the output error convergence rate by adjusting controller parameters. As a consequence,

$$\begin{aligned}\lim_{t\to\infty}\|\mathbf{y}\| \le & \sqrt{\frac{12}{\omega\underline{\lambda}(\mathbf{P})\underline{\lambda}(\mathbf{Q}_o)}} \sup_{t_0<\tau<t}\|\boldsymbol{\varepsilon}_\sigma(\tau)\| \\ & + \sqrt{\frac{\overline{\kappa}_\sigma}{\omega\underline{\lambda}(\mathbf{P})}Tr(\mathbf{W}_\sigma^T\mathbf{W}_\sigma)}. \end{aligned} \tag{82}$$

Considering the Frobenius norm definition, one can obtain the following bounds for the weighting vector $\tilde{\mathbf{W}}_\sigma$.

$$\begin{aligned}\lim_{t\to\infty}\|\tilde{\mathbf{W}}_\sigma\|_F \le & \sqrt{\frac{12}{\omega\underline{\lambda}(\boldsymbol{\Gamma}_\sigma)\underline{\lambda}(\mathbf{Q}_o)}} \sup_{t_0<\tau<t}\|\boldsymbol{\varepsilon}_\sigma(\tau)\| \\ & + \sqrt{\frac{\bar{\kappa}_\sigma}{\omega\underline{\lambda}(\boldsymbol{\Gamma}_\sigma)}Tr(\mathbf{W}_\sigma^T\mathbf{W}_\sigma)}. \end{aligned} \tag{83}$$

This completes the transient performance analysis.

5. NUMERICAL EVALUATION OF THE DESIGNED CONTROLLER

To test the proposed controller, a cooperative robot arm consisting of two manipulators with 3 degree of freedom is considered. More details about the model of robot/object, and their parameters are given in [41]. Consider

$$\boldsymbol{\tau}_{d,i}(t) = \begin{bmatrix} 0.4\cos(2t) \\ 0.1\sin(t)+\cos(2t) \\ 0.2\sin(t) \end{bmatrix},\ i=1,\ 2, \tag{84}$$

as external disturbance, which is imposed on both manipulators. The desired trajectory is given by

$$\mathbf{x}_{od}(t) = \begin{bmatrix} \frac{0.5\cos(t+4)}{1+\sin(t)^2} \\ 1.65+\frac{0.5\sin(t)\cos(t)}{1+\sin(t+4)\sin(t)} \\ 0 \end{bmatrix}. \tag{85}$$

The desired value of $\mathbf{F}_{Id}(t)$ is zero. In this simulation, assume that the initial value of $\mathbf{q}$ is $\mathbf{q}(0) = [-0.23\ \ -2.3\ \ 2.52\ \ -2.52\ \ 2.3\ \ 3.4]^T$. The numerical values of the controller-observer structure are $l_v = 45$, $\gamma = 5$, $k_v = 15$, $\mu = 10$ and $\mathbf{K}_I = 100\mathbf{I}_6$. To approximate the function $\boldsymbol{\sigma}(t)$, the first 11 terms of the q-analogue of the Bernstein-Schurer-Stancu operator are chosen. The initial values of the estimated parameters are set to zero. The matrix of learning rates is selected as $\boldsymbol{\Gamma}_\sigma = 10^{-5}\times\mathbf{I}_{33}$. Suppose that the estimation error is negligible. For the non-observer-based control strategy, all simulation settings are the same as the observer-based one, except that the velocity signal is accessible. Under these settings, the desired path and system response are plotted in Fig. 1. As can be seen in this figure, the controller is successful in tracking the reference trajectory in the presence of external disturbances and uncertainties. Motor voltages as control efforts are given in Figs. 2 and 3. These figures show that motor volt-

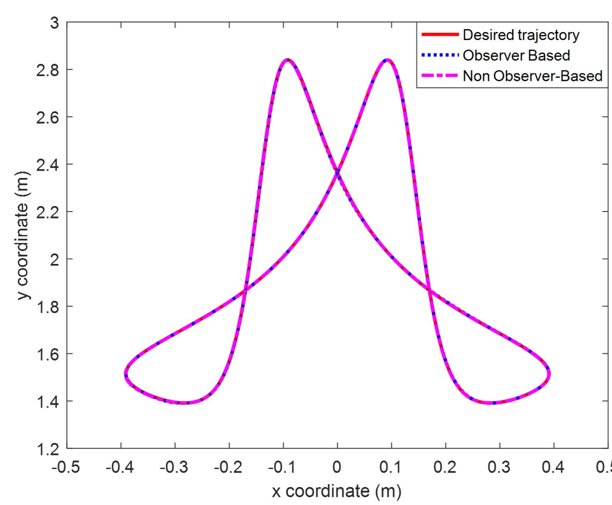

Fig. 1. The object trajectory tracking in the XY plane.

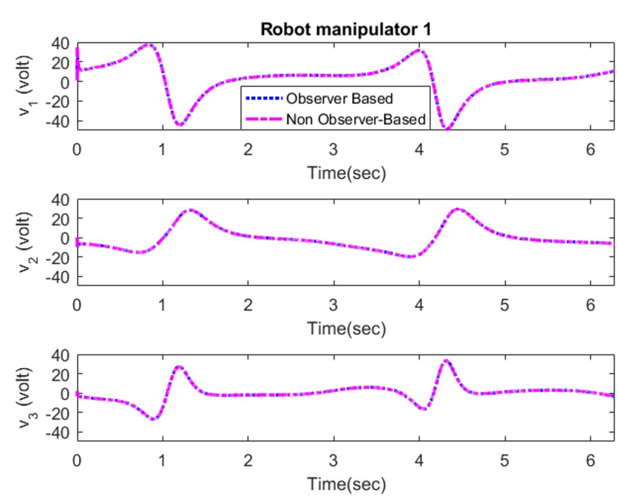

Fig. 2. Control signals of the first manipulator.

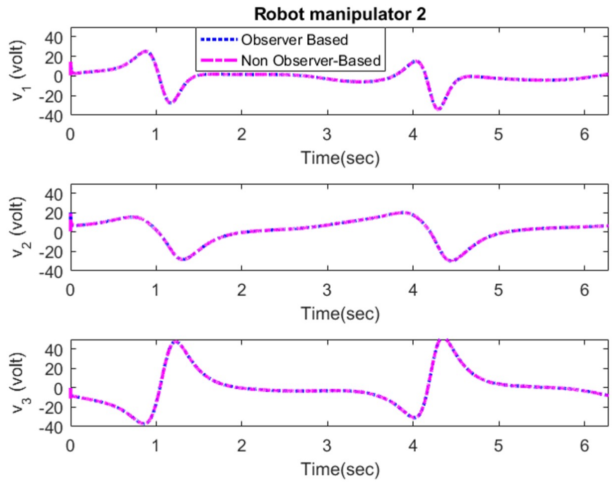

Fig. 3. Control signals of the second manipulator.

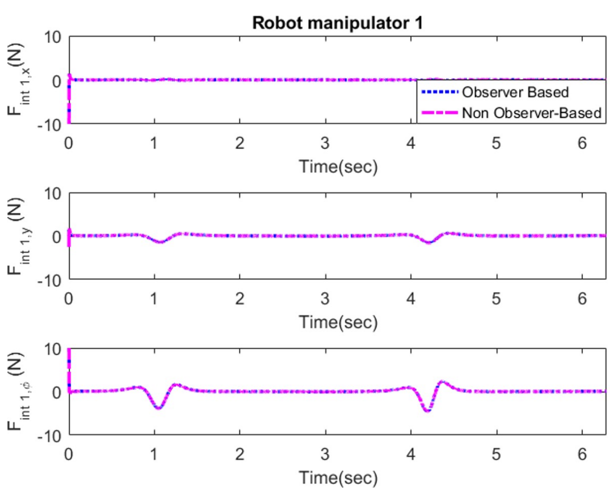

Fig. 4. The internal force for the first manipulator.

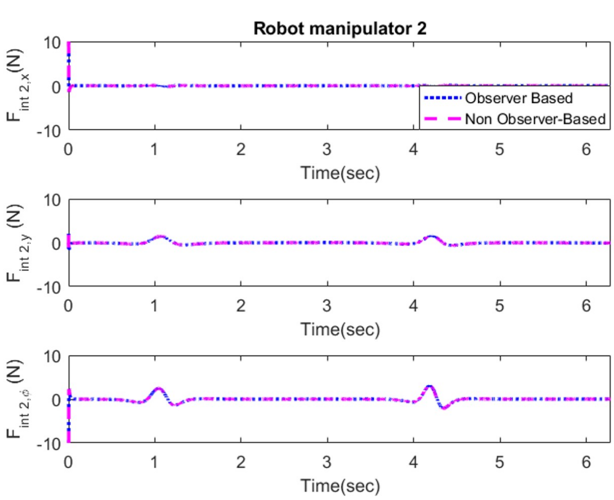

Fig. 5. The internal force for the second manipulator.

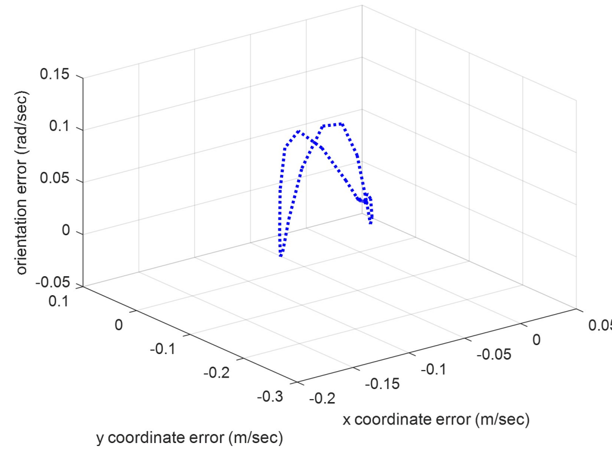

Fig. 6. The velocity estimation error for the observer-based approach.

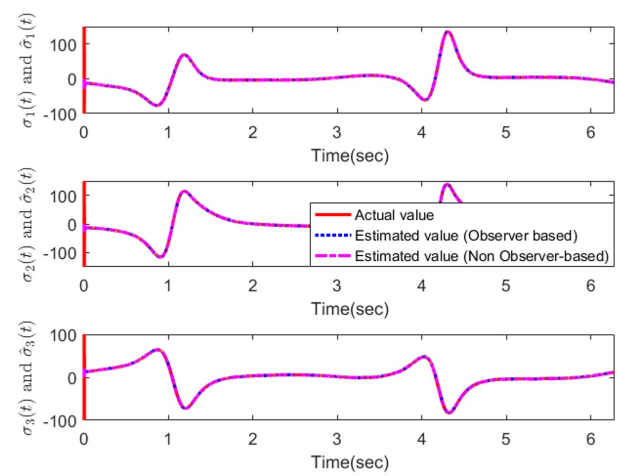

Fig. 7. The estimation performance of $\boldsymbol{\sigma}(t)$.

ages are reasonable. For the internal forces, the tracking performances are presented in Figs. 4 and 5. The observer error in the calculation of the velocity of the COM of the object is shown in Fig. 6. The estimated uncertainties are also given in Fig. 7.

6. CONCLUSION

In this paper, an observer-based controller has been introduced for cooperative robots using the q-analogue of the Bernstein-Schurer-Stancu operator as an alternative for fuzzy systems and neural networks. Both the controller and observer are model-free. In order to highlight the satisfactory performance of the observer, a comparison with the non-observer-based controller was presented. Simulation results show the high capabilities of the proposed scheme in performing complex tasks. Indeed, simulation results showed the considerable influence of this estimator in enhancing the controller performance.

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